Two-Level Value Function Approach to Nonsmooth Optimistic Bilevel Programs

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Abstract

The optimistic (pessimistic) bilevel optimization problem can be modelled as minimizing the optimistic (resp. pessimistic) two-level optimal value function:

$$\varphi_o(x) := \min_y\{F(x,y) : y \in \Psi(x)\} \to \min_{x\in K},$$

resp.

$$\varphi_p(x) := \max_y\{F(x,y) : y \in \Psi(x)\} \to \min_{x\in K},$$

where

$$\Psi(x) = \{y : g(x,y) \leq 0,\ f(x,y) \leq \varphi(x)\}$$

denotes the set of optimal solutions and $\varphi(x) = \min_y\{f(x,y) : g(x,y) \leq 0\}$ is the optimal value function of the lower level problem. Topic of the presentation is the formulation of necessary optimality conditions for the optimistic problem using tools from variational analysis [2] in the case when the bilevel optimization problem is formulated with nonsmooth data. This generalizes the necessary optimality conditions for continuously differentiable optimistic bilevel optimization problems presented in [1].

Keywords: optimistic and pessimistic bilevel programming, two-level value functions, variational analysis, generalized differentiation, optimality conditions.

References

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[2] B.S. Mordukhovich, *Basics of Variational Analysis and Applications,* Springer, New York, 2018.

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